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Non-Biased Interval Estimation of LESLIE's Removal Method

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Abstract

LESLIE's removal method, which is used to estimate the initial population size and the removal ratio simultaneously, is modeled in terms of the product of binomial distributions. The approximation of these binomial distributions to the standard normal distribution presents a new method which has no bias for estimators. This is an improvement over the maximum likelihood method and the likelihood ratio test, and is essentially equivalent to a standard goodness-of-fit test. The confidence region on the 2-dimensional plane defined by the initial population size and the removal ratio gives each confidence interval. This region is defined by the chi-square distribution with 2 degrees of freedom.

Key words LESLIE's method, removal method, bias, likelihood ratio test, goodness-of-fit test

Introduction

SEBER (1982) reviewed catch-effort methods for a closed population : LESLIE's method and the removal method. The former is based on the regression model, which is successful for point estimation. However, the interval estimation is mistaken because the regression model is not adequate for LESLIE's method. The main reason is that each catching data is not independent. On the other hand, the latter is based on binomial distribution, which is a better model for the interval estimation. However, it was a special model in which all sampling efforts are equal. Therefore, it has little application in practice. In this paper, we present a new method based on the extended removal method. It has large application and no bias.

ZIPPIN (1956) made a great contribution to the removal method. He applied the maximum likelihood method for the point estimation by the graphical method. For the interval estimation, he used asymptotic variance. Therefore, his confidence intervals are symmetrical around the point estimation. He also used a standard goodness-of-fit statistic

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to test the validity of the model, and showed that it is asymptotically equivalent to the objective function in this paper. However, his model was the removal method and had little application.

SCHNUTE(1983) applied the maximum likelihood method for the point estimation and the likelihood ratio test for the interval estimation. This method is more useful than ZIPPIN's. However, his model was also the removal method and the maximum likelihood method has a bias. The most serious mistake of his method is that he used the chi-square distribution with 1 degree of freedom. The degree of freedom must be 2. This will be mentioned strictly in this paper.

AKAMINE(1990) applied the maximum likelihood method and the likelihood ratio test to the extended removal method in which sampling efforts are not equal. In this paper, "LESLIE's removal method" means this model, which has large application, not a regression model. He corrected the degree of freedom of the chi-square distribution to 2. Additionally, he used the approximation of a binomial distribution to the standard normal distribution, and checked the confidence region for each sampling time.

We develop AKAMINE's(1990) method and correct the bias of the maximum likelihood method. It is essentially equivalent to the weighted least-squares method and the goodness-of-fit test. We use MARQUARDT's(1963) method which is one of optimization to obtain point point estimators and an easy method to calculate confidence intervals.

Model

The stochastic model of LESLIE's removal method is defined as follows (SEBER 1982) :

$$L=\prod_{i=1}^{m} P_i, \tag{1.1}$$

$$P_i=\frac{n_i!}{r_i!\ (n_i-r_i)!}\ p_i^{r_i}\ (1-p_i)^{\ n_{i+1}}, \tag{1.2}$$

$$n_i=n-R_{i-1}, \quad R_i=\sum_{k=1}^{i} r_k, \tag{1.3}$$

$$p_i=x_i\ p. \tag{1.4}$$

Where n : initial population size,
p : removal ratio,
m : sample size,
n_i : population size of the i th sample,
p_i : removal ratio of the i th sample,
r_i : size of the i th sample removed from the population,
x_i : units of effort expanded on the i th sample.

The value of likelihood L is a product of P_i (probability of the binomial distribution).

The essence of this model is

$$E(r_i)=n_i\ p_i. \tag{1.5}$$

Where $E(\theta)$: expected value of θ.

Substitution of (1.3) and (1.4) into (1.5) gives

$$E\left(\frac{r_i}{x_i}\right) = np - pR_{i-1}. \tag{1.6}$$

Letting $Y = r_i/x_i$ and $X = R_{i-1}$, we obtain the regression model called "LESLIE's method" or "DELURY's method". This method is useful for point estimation. However, it is difficult to obtain the confidence intervals of p and n by this method. The main reason is that each data ($R_i = \Sigma r_k$) is not independent. This does not satisfy the supposition of the regression model. For point estimation, this has no influence because it is based on only the least -squares method and not another regression theory. However, interval estimation is influenced by this greatly. In general, the following variance is used for interval estimation:

$$\sigma^2 = \frac{S^2}{m-2}. \tag{1.7}$$

Where S^2 : residual sum of squares.

For example, when the sampl size is only 2 ($m=2$), this regression model has no interval. Therefore, we must use another variance.

Finally, we use the following objective function in this paper:

$$Y = \sum_{i=1}^{m} \frac{(r_i - n_i\, p_i)^2}{n_i\, p_i(1-p_i)}. \tag{1.8}$$

This is a nonlinear model for parameters (p, n) and regarded as the weighted least-squares model of (1.5) with the following weight:

$$\sigma^2 = n_i\, p_i(1-p_i). \tag{1.9}$$

This is the variance of binomial distribution (1.2). In this paper, we will obtain (1.8) by correcting the maximum likelihood method.

For correct interval estimation, we must use (1.1) directly. Henceforth we use the expression "LESLIE's removal method" to mean (1.1) and not (1.6). We do not use the regression model (1.6) in this paper. Although ZIPPIN (1956) applied the maximum likelihood method to (1.1) directly, his graphical method is not useful because his model is based on the removal method and all sampling efforts x_i must equal 1. SCHNUTE (1983) also applied the maximum likelihood method to the removal method. He used the simplex method for the optimization of (1.1) using a computer and suggested that the maximum likelihood estimators have bias. Although he applied the likelihood ratio test to obtain the interval estimation of n, he used the chi-square distribution with 1 degree of freedom. He used the following curve as a parameter space:

$$\frac{\partial L}{\partial p} = 0. \tag{1.10}$$

This equation means the function $p = f(n)$ in which p takes the value to make L be maximum at each n. This is opposed to independency of p and n. Therefore, he made a mistake for the degree of freedom.

AKAMINE (1990) also applied the likelihood ratio test to (1.1) directly by using the chi -square distribution with 2 degrees of freedom. He also used the (1.10) curve but for convenience. On the other hand, he used the transformation

$$z_i=\frac{r_i-n_i\,p_i}{n_i\,p_i(1-p_i)} \tag{1.11}$$

to calculate confidence regions on the $(p,\ n)$-plane at each sampling time. This is based on the "DE MOIVRE - LAPLACE limit theorem" (MOOD *et al.* 1974) which proves that the normal distribution

$$N_i=\frac{1}{\sqrt{2\pi\ n_i\ p_i(1-p_i)}}\exp\left(-\frac{1}{2}z_i^2\right) \tag{1.12}$$

approximates the binomial distribution P_i when

$$n>30,\ np>5,\ n(1-p)>5,$$

and is usually satisfied for LESLIE's removal method. Therefore, AKAMINE (1990) based his analysis on the following model instead of (1.1).

$$L=\prod_{i\ 1}^{m}N_i. \tag{1.13}$$

Henceforth we use (1.13) because it is almost equivalent to (1.1) and easier to treat for interval estimation.

Non-Biased Estimation

The likelihood ratio test is defined as follows: The null hypothesis is

$$\mathrm{H}_0\ :\ \theta_1=\theta_{10},\quad \cdots\cdots,\quad \theta_n=\theta_{n0}. \tag{2.1}$$

Where n: number of parameters to test,

θ_{i0}: i th parameters, constant under the null hypothesis.

The likelihood ratio λ is defined by

$$\lambda=L_0/L_{\max}. \tag{2.2}$$

Where L_0 : likelihood of H_0,

$L_{\max}$: maximum value of likelihood.

The distribution of $(-21\mathrm{n}\lambda)$ asymptotically approximates the chi-square distribution with n degrees of freedom as the number of observations increases. Therefore, interval estimators can be obtained from the likelihood ratio. The point estimators, which give $L_{\max}$, are obtained by the optimization.

However, it is well-known that these values have bias. The famous example is the estimators of the normal distribution $N(\mu,\ \sigma^2)$. The estimator of variance from the maximum lieklihood method is larger than "unbiased variance". For LESLIE's removal method, SCHNUTE (1983) showed a biased sample (See Example 1), and AKAMINE (1990) showed that when one of the parameters is obvious, another estimator has no bias. Although this bias is small in practice, no biased estimator is better than biased one especially for point estimation.

We apply the likelihood ratio test to (1.13) directly. Where $(\theta_1,\ \theta_2)=(p,\ n)$. The logarithm of (1.12) is

$$\ln N=-\frac{1}{2}\ln 2\pi-\frac{1}{2}\ln np(1-p)-\frac{1}{2}z^2. \tag{2.3}$$

Where the subscripts i are omitted. Substitution of (2.3) into the logarithm of (2.2)

$$-21\mathrm{n}\lambda=-2\ (\ln L_0-\ln L_{\max})$$

gives

$$-2\ln\lambda=\sum_{i}^{m}\ln\frac{n_0\ p_0(1-p_0)}{n^*p^*(1-p^*)}+\sum_{i}^{m}z_0{}^2-\sum_{i}^{m}z^{*2}. \tag{2.4}$$

Where L_0 is evaluated at $(p_0,\ n_0)$ and $L_{\max}$ at $(p^*,\ n^*)$. The first term on the right side is almost 0 in the neighborhood of the point estimator, and contributes less than the other terms on the far side. In the other terms, each z_i is distributed as the standard normal distribution $N(0,1)$. Thus the second term on the right side is distributed as a chi-square distribution with m degrees of freedom. This is the definition of the chi-square distribution. The third term on the right side is distributed as a chi-square distribution with $(m-2)$ degrees of freedom, because p and n have been estimated. Therefore, $(-2\ln\lambda)$ is distributed as a chi-square distribution with 2 degrees of freedom. Although the condition of the number of observations (m) is important for the likelihood ratio test in general, it is not so important for LESLIE's removal method for this reason.

The first term on the right side is not 0 exactly. It seems that this is the cause of bias for the maximum likelihood method, and it is useful to omit this term. Therefore, the objective function Y is given by

$$Y=\sum_{i=1}^{m}z_i{}^2=\sum_{i=1}^{m}\frac{(r_i-n_i\ p_i)^2}{n_i\ p_i(1-p_i)}. \tag{2.5}$$

The minimum Y is the non-biased point estimator of $(p,\ n)$ from (1.5). This equation is a weighted least-squares version of (1.5). ZIPPIN (1956) had already obtained this equation as a goodness-of-fit statistic. Iteration methods are necessary to solve this objective function because this is a nonlinear model for p and n. For the interval estimation, the null hypothesis

$$\mathrm{H}_0\ :\ p=p_0,\ n=n_0 \tag{2.6}$$

gives the confidence region on the $(p,\ n)$-plane as follows:

$$Y-Y_{\min}\leqq\chi^2(2). \tag{2.7}$$

This expression indicates the probability that the true values of $(p,\ n)$ exist in this region.

On the other hand, AKAMINE (1990) obtained the region as the product set of $|z_i|\leqq1.96$ $(i=1\sim m)$ on the $(p,\ n)$-plane. This region is completely covered by the following region derived from the second term of the right side of (2.4).

$$Y\leqq\chi^2(m). \tag{2.8}$$

This expression gives the probability that $\boldsymbol{r}$ is obtained from each $(p,\ n)$ in this region. While (2.7) is a relative relation and gives confidence intervals, (2.8) is an absolute relation and judges the condition of dara $\boldsymbol{r}$. Therefore, the region of (2.8) is very wide for good conditional data, and cannot exist for a bad one. For practical purposes, a more useful relation to judge the condition of data is given by

$$Y_{\min}\sim\chi^2(m-2). \tag{2.9}$$

This is the third term of the right side of (2.4).

This way is essentially equivalent to a standard goodness-of-fit statistic. For the removal method, ZIPPIN (1956) suggested that (2.5) is asymptotically equivalent to the equation

$$T=\sum_{i=1}^{m}\frac{(r_i-ns_i)^2}{ns_i}+\frac{(n-R_m-nt)^2}{nt}. \tag{2.10}$$

Where $s_i=p_i\prod_{k=1}^{i-1}(1-p_k)$, $t=\prod_{k=1}^{m}(1-p_k)$.

Therefore, it is also useful to define the objective function as T instead of Y, because the likelihood ratio test is essentially equivalent to a standard goodness-of-fit test.

An explanation of omitting the first term on the right side of (2.4) is as follows: This operation is equal to transforming a normal distribution (1.12) to the standard normal distribution

$$N(0,1)=\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}z^2\right). \tag{2.11}$$

Namely, all variances $np(1-p)$ of each sampling time are assumed to be equal. For (2.11), likelihood (probability) and z are one-to-one correspondence. In this case, the objective function (2.5) is also a function of the likelihood of $N(0,1)$. The simple example, $m=1$ and $r=100$, helps us to understand this explanation. Which is a better estimation for this example: A $(p, n)=$ (0.1, 1000) or B (0.01, 10000)? Likelihood (1.12) judges that A is better than B because variance of A (90) is smaller than that of B (99). Namely, in the case of B the probability of getting some value $r\neq 100$ is larger than that of A. On the other hand, likelihood (2.11) judges that A is equal to B exactly. The essence of LESLIE's removal method is $E(r)=np$, and data is r not A or B. Therefore, it is natural to judge that A is equal to B. This explanation justifies the operation to omit the first term of (2.4).

Optimization and Interval Estimation

1. MARQUARDT's method

This method is one of optimization to obtain the minimum value of the objective function Y. If the model (2.5) is linear, the solution is easily obtained by solving the following simultaneous equations:

$$\frac{\partial Y}{\partial n}=0, \qquad \frac{\partial Y}{\partial p}=0. \tag{3.1}$$

However, the model (2.5) is nonlinear, and it is difficult to solve these nonlinear equations. For a nonlinear model, it is better to search for the minimum point of Y directly by using optimization methods.

For the model (2.5), it is easy to find the solution because the number of parameters is only 2. SCHNUTE(1983) used the simplex method and AKAMINE(1990) used NEWTON's method for model (1.1). In this paper, we present a BASIC program using MARQUARDT's (1963) methhod. This program searches for the minimum point of (2.5) quickly with a wide region of convergence. Therefore, we can easily obtain the point estimators of LESLIE's removal method.

MARQUARDT's method is a mixture method of the steepest descent method and NEWTON's method. For searching for the minimum point, it is written as the simultaneous equations

$$(\boldsymbol{H}+\lambda\boldsymbol{I})\ \Delta\theta=-\boldsymbol{g}. \tag{3.2}$$

Where $\boldsymbol{H}$: Hessian matrix,
λ : reducing factor,
$\boldsymbol{I}$: unit matrix,
$\Delta\theta$: correction vector of parameters,
$\boldsymbol{g}$: gradient vector.

First, let $\lambda=1$. This is the steepest descent method. Next, iterate solving (3.2) with controlling λ. When Y becomes smaller, let λ be small. On the other hand, when Y becomes larger, let λ be large and repeat the same iteration. Finally, let $\lambda \to 0$. This is NEWTON's method. Although λ is added to the diagonal elements of $\boldsymbol{H}$, it is equivalent to multiplying the diagonal elements of $\boldsymbol{H}$ by $(1+\lambda)$ because of the scaling of parameters.

In the model (2.5), concrete expressions of (3.2) are as follows :

$$\boldsymbol{H}=\begin{pmatrix} \dfrac{\partial^2 Y}{\partial n^2} & \dfrac{\partial^2 Y}{\partial n \partial p} \\ \dfrac{\partial^2 Y}{\partial n \partial p} & \dfrac{\partial^2 Y}{\partial p^2} \end{pmatrix}, \quad \boldsymbol{g}=\begin{pmatrix} \dfrac{\partial Y}{\partial n} \\ \dfrac{\partial Y}{\partial p} \end{pmatrix} \tag{3.3}$$

$$\frac{\partial Y}{\partial n}=-\Sigma\frac{(r-np)(r+np)}{n^2p(1-p)} \tag{3.4}$$

$$\frac{\partial Y}{\partial p}=-\Sigma x\frac{(r-np)(r+np-2rp)}{np^2(1-p)^2} \tag{3.5}$$

$$\frac{\partial^2 Y}{\partial n^2}=2\Sigma\frac{r^2}{n^3p(1-p)} \tag{3.6}$$

$$\frac{\partial^2 Y}{\partial n \partial p}=\Sigma x\frac{r^2(1-2p)+n^2p^2}{n^2p^2(1-p)^2} \tag{3.7}$$

$$\frac{\partial^2 Y}{\partial p^2}=2\Sigma x^2\frac{r^2(1-3p+3p^2)+np^3(n-2r)}{np^3(1-p)^3} \tag{3.8}$$

Where subscripts i of the right sides of (3.4) ~ (3.8) are omitted.

The process of MARQUARDT's method is as follows :

(a) Input data $\boldsymbol{r}$, $\boldsymbol{x}$ and initial values of θ^{old}.
(b) Calculate Y^{old}.
(c) Let $\lambda=1$.
(d) Calculate $\boldsymbol{H}$ and $\boldsymbol{g}$ by using (3.3) ~ (3.8).
(e) Multiply the diagonal elements of $\boldsymbol{H}$ by $(1+\lambda)$.
(f) Solve the simultaneous equations (3.2).
(g) $(\theta^{\text{old}}+\Delta\theta)\to\theta^{\text{new}}$.
(h) Calculate Y^{new}.
(i) If $Y^{\text{new}}<Y^{\text{old}}$, then let $\lambda/2\to\lambda$, $\theta^{\text{new}}\to\theta^{\text{old}}$, $Y^{\text{new}}\to Y^{\text{old}}$ and go to (d).
(j) If $Y^{\text{new}}\geqq Y^{\text{old}}$, then let $\lambda*2\to\lambda$ and go to (e).
(k) If $Y^{\text{new}}\geqq Y^{\text{old}}$ after 10 times iterations of (j) continuously, judge it to be the solution and end this program.

An example BASIC program is shown in Appendix. The values estimated by the regression method are the best initial values of parameters for good convergence. It

converges in about 20 iterations usually.

2. Interval estimation

We can obtain the interval estimators by drawing contours of (2.7) on the (p, n)-plane. Although it is difficult to draw contours by freehand, it is easy now by using micro-computers and X-Y plotters. Many algorithms have been developed for drawing contours. In this paper, we use the "grid method" explained in the following paragraphs. This is a simpl and popular algorithm to draw contours and sufficient to obtain the interval estimators.

The process of this method drawing a contour $Y(x, y)=\alpha$ on the (x, y)-plane is as follows:

(a) Let $F(x, y)=Y(x, y)-\alpha=0$.

(b) Determine the drawing range as $x=x_1\sim x_2$, $y=y_1\sim y_2$.

(c) Make $m\times n$ grids in this range as $x_k=x_1+sk$, $s=(x_2-x_1)/m$, $y_h=y_1+th$, $t=(y_2-y_1)/n$.

(d) For each grid, let $\boldsymbol{p}_1=(x_k, y_h)$, $\boldsymbol{p}_2=(x_k, y_{h+1})$, $\boldsymbol{p}_3=(x_{k+1}, y_{h+1})$, $\boldsymbol{p}_4=(x_{k+1}, y_h)$, $\boldsymbol{p}_5=\boldsymbol{p}_1$. When $F(\boldsymbol{p}_i)F(\boldsymbol{p}_{i+1})\leqq 0$, calculate $\boldsymbol{q}_i=(x, y)$ as follows:

$$\boldsymbol{q}_i=\frac{F(\boldsymbol{p}_i)\boldsymbol{p}_{i+1}-F(\boldsymbol{p}_{i+1})\boldsymbol{p}_i}{F(\boldsymbol{p}_i)-F(\boldsymbol{p}_{i+1})}. \tag{4.1}$$

This is the approximated cross point of the grid and a contour. In general, there are only 2 cross points $\boldsymbol{q}_i$ and $\boldsymbol{q}_k$ on the grid. In this case, draw the line $\boldsymbol{q}_i\boldsymbol{q}_k$.

(e) Specially, there are 4 cross points $\boldsymbol{q}_1\sim\boldsymbol{q}_4$ on the grid. In this case, only two cases are possible as lines $\{\boldsymbol{q}_1\boldsymbol{q}_2, \boldsymbol{q}_3\boldsymbol{q}_4\}$ or $\{\boldsymbol{q}_1\boldsymbol{q}_4, \boldsymbol{q}_2\boldsymbol{q}_3\}$. In model (2.5), only the latter case is possible. Therefore, draw the two lines $\boldsymbol{q}_1\boldsymbol{q}_4$ and $\boldsymbol{q}_2\boldsymbol{q}_3$.

An example BASIC program of this method, which draws equilibrium yield contours of the BEVERTON-HOLT model on a CRT display, is shown in KATO(1988).

The demerits of this method are as follows:

(1) It requires much time to draw.

(2) For approximation (4.1), $\boldsymbol{q}$ is not such a good approximation to the real cross point of the grid and a contour.

As an example of (1), the 150×100 grids (Figs. 1, 4, 5) take 30 minutes to draw with our PC-9801VX (NEC) micro computer. On the other hand, the 50×30 grids (Figs. 2, 3) take only 3 minutes. As an example of (2), the drawn contour is zigzagged and its edges are not so sharp. These narrow and sharp figures are difficult to draw accurately by any algorithm.

The most countermeasure for demerit (2) is making grids small. However, this aggravates demerit (1). For practical purposes, the figure of a whole contour is not so necessary. For the interval estimation, magnified figures of both edges are useful. These figures are drawn sharply and rapidly with large grids. Therefore, demerits (1), (2) and operation (e) are not so concerned with the interval estimation.

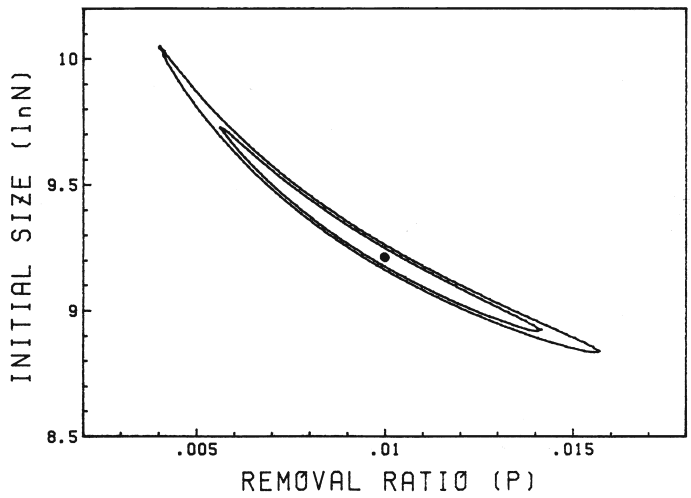

Fig. 1. The confidence region on the (p, n)-plane for Example 2. The white circle represents point estimators and the black circle represents true values. In this figure, the two circles almost overlap.

Examples

For the check of our method, the artificial data made from the model (1.1) is useful. Therefore, we examine the data of SCHNUTE(1983) and AKAMINE(1990).

〔Example 1〕 Point estimation for SCHNUTE's(1983) data

This model is $\boldsymbol{x}=(1, 1, 1)$, and data is $\boldsymbol{r}=(90,60,40)$. The true values of parameters are $(p, n)=(1/3, 270)$ obviously and this data has no error. Although regression method (1.6) gives the true values, the maximum likelihood method gives biased values (0.341807, 265.255) (SCHNUTE 1983; AKAMINE 1990). The BASIC program in the Appendix with initial values (0.3, 300) gives the true values (0.333333, 270.000) after 11 iterations. Therefore, the objective function (2.5) is better than the maximum likelihood method.

〔Example 2〕 AKAMINE's(1990) data-1

This model is $\boldsymbol{x}=(7, 5, 10, 8, 4)$, and data is $\boldsymbol{r}=(700,465,884,636,293)$. The true values are $(p, n)=(0.01, 10000)$ and $\boldsymbol{r}$ has a little rounding error. The BASIC program with the initial values (0.02, 10000) gives the solution (0.00998152, 10018.6) after 17 iterations. These values are nearer to the true values than the maximum likelihood solution (0.01003895, 9968.41) (AKAMINE 1990). The latter has more error caused by bias. The BASIC program also gives $Y=0.0000505631$ at the solution. This is $Y_{\min}$ and much smaller than $\chi^2(3)_{0.975}=0.2158$. The test of (2.9) shows that this data is too high in precision to have been obtained from model (1.1) naturally.

The confidence regions of (2.7) and (2.8) on the (p, n)-plane are shown in Fig. 1 drawn by the grid method. The outer contour is $Y=\chi^2(5)_{0.05}=11.07$ from (2.8), in this region we obtain data $\boldsymbol{r}$ in 95%. The inner contour is $Y=\chi^2(2)_{0.05}=5.991$ ($Y_{\min}=0$) from (2.7), this region involves true (p, n) in 95%. The upper left and lower right parts of the

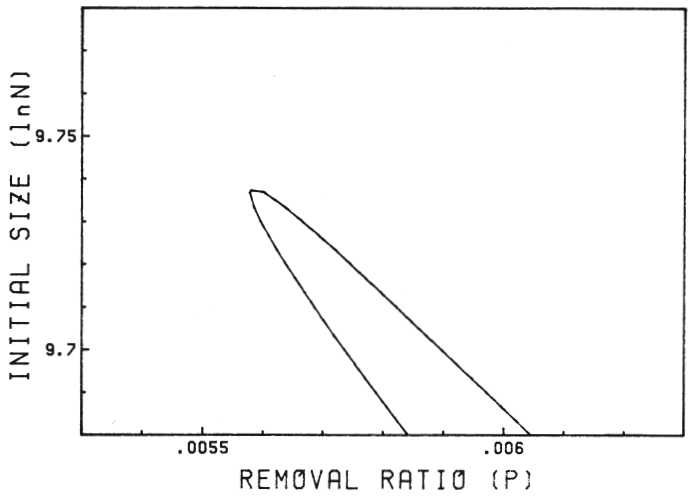

Fig. 2. The magnified figure of the upper left part of Fig. 1.

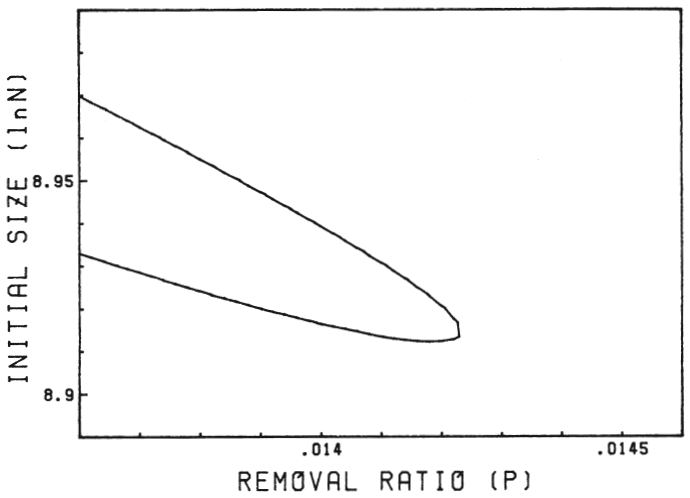

Fig. 3. The magnified figure of the lower right part of Fig. 1.

inner contour are magnified to be shown in Figs. 2 and 3. From these figures we can read 95% confidence intervals as $n=7420\sim16933$ and $p=0.00558\sim0.01423$. The upper part of n is much wider then the lower part. AKAMINE (1990) gave 95% confidence intervals as $n=7411\sim16611$ and $p=0.0057\sim0.0143$ by using the maximum likelihood method. These are almost equal to the new method in practice.

〔Example 3〕 AKAMINE's (1990) data-3

This data has more error then Example 2 as $\boldsymbol{r}=(736,488,827,636,290)$. The point estimations are (0.0119783, 8575.14) and $Y_{\min}=5.35061$, which is smaller than $\chi^2(3)_{0.05}=7.815$. Therefore, this data is obtained in 95% from model (1.1). The confidence region

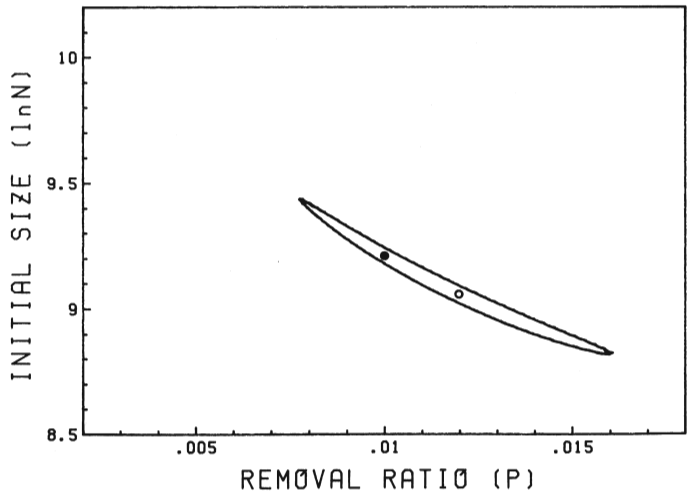

Fig. 4. The confidence region on the (p, n)-plane for Example 3. The white circle represents point estimators and the black circle represents true values.

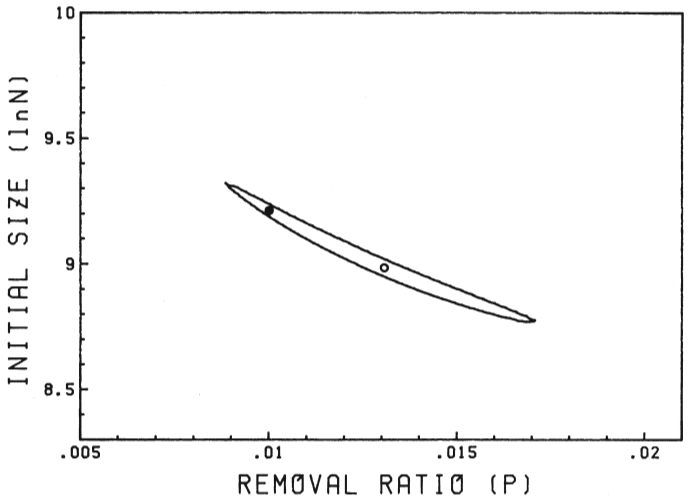

Fig. 5. The confidence region on the (p, n)-plane for Example 4. The white circle represents point estimators and the black circle represents true values.

is shown in Fig. 4. The contours of (2.8) ($Y=11.07$) and (2.7) ($Y=5.35+5.99=11.34$) are almost overlapped. The magnified figures, which give confidence intervals as $n=6714 \sim 12632$ and $p=0.00769 \sim 0.01612$, are omitted in this paper. AKAMINE (1990) gave point estimations as (0.01202092, 8543.93) and interval estimations as $n=6697 \sim 12632$ and $p=0.0077 \sim 0.0161$. Although interval estimations are almost equal to the new method, point estimation has more error caused by bias.

〔Example 4〕

This data has more error than Example 3 as $\boldsymbol{r}=(754,500,799,636,287)$. The point estimations are (0.0130701, 7976.33) and $Y_{\min}=11.8916$, which is larger than 7.815. Therefore, it is impossible to obtain this data in 95% from model (1.1). The value of $Y_{\min}$ is larger then 11.07. Therefore, the region of (2.8) does not exist. The following analysis is meaningless because this data is not according to model (1.1). Fig. 5 shows only the confidence region of (2.7) in which true values still exist. The magnified figures, which give 95% confidence intervals as $n=6393\sim11181$ and $p=0.00884\sim0.01716$, are omitted in this paper.

Conclusion

The standard procedures for LESLIE's removal method is as follows:

(1) The BASIC program in the Appendix gives the point estimators (p, n) and $Y_{\min}$.

(2) The conditon of data is tested by the equation

$$Y_{\min}\sim\chi^2(m-2).$$

(3) The confidence region on the (p, n)-plane is obtained by the equation

$$Y=Y_{\min}+\chi^2(2).$$

The upper left and lower right parts of this contour give the confidence intervals of p and n. This contour is easily drawn by using a BASIC program according to KATO (1988).

In the procedure (2) if the test is significant, the data is not according to the model (1.1) and the other procedures are meaningless. Making the confidence level of the procedure (3) equal to that of the procedure (2) is one way to avoid this difficulty. However, it is more important to obtain good conditional data, according to model (1.1), through a well-designed investigation or experiment.

Although these procedures have no bias caused by the maximum likelihood method, the values of estimations are not so different from the latter ones in practice. Procedure (2), which tests the condition of data and models, is the most important in this analysis. It is essentially equivalent to a standard goodness-of-fit test.

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除去法における LESLIE 法の偏りのない区間推定

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LESLIE 法は初期資源尾数と除去率を同時に推定する方法であるが，二項分布の積としてモデル化される．この二項分布を標準正規分布に近似することにより，偏りのない推定方法が導かれる．この方法は最尤法および尤度比検定の改良であり，本質的には適合度検定と同一のものである．初期資源尾数と除去率の2次元平面に描いた信頼域よりそれぞれの信頼区間が求まる．この信頼域は自由度2の χ^2 分布より決定される．

Appendix

This BASIC program searches for the minimum point of the objective function $Y=\Sigma z^2$ by MARQUARDT's method. It involves the non-biased point estimators of LESLIE's method. Users input data after 30,000 lines; the type of variable r is an integer because it is a number of individuals. When a user inputs a very large number for r, he must exchange all "IR" of variables in this program list to "R" because BASIC cannot treat a large integer.

```
10 '-----------------------------------------------
20 '     LESLIE's (DELURY's) method
30 '          MARQUARDT's method for Sigma z*z
40 '                              by TATSURO AKAMINE
50 '                                   1990/1/20
60 '-----------------------------------------------
1010  GOSUB *INIT1
1015  GOSUB *PRIOUT
1020  AMBDA=1 : BNU=2
1030  READ POLD,ANOLD
1035  IF ANOLD<IR2(M) THEN PRINT "n is small" : STOP
1038  K1=0
1040 *REPEAT1
1045  IF K1>50 GOTO *END2
1050  FOR I=1 TO M
1060     P(I)=POLD*X(I)
1070     AN(I)=ANOLD-IR2(I-1)
1080  NEXT I
1100  GOSUB *YVALUE
1110  YOLD=Y
1115  GOSUB *OUTPUT
1200  GOSUB *DIFFER
1210  K2=0
1220 *REPEAT2
1250  A=A*(1+AMBDA) : D=D*(1+AMBDA)
1300  GOSUB *SOLUB
1400  PNEW=POLD+DELTAP : ANNEW=ANOLD+DELTAN
1450  FOR I=1 TO M
1460     P(I)=PNEW*X(I)
1470     AN(I)=ANNEW-IR2(I-1)
1480  NEXT I
1490  GOSUB *YVALUE
1500  YNEW=Y
1510  IF YNEW<YOLD THEN AMBDA=AMBDA/BNU : K1=K1+1
      : POLD=PNEW : ANOLD=ANNEW : GOTO *REPEAT1
1520     AMBDA=AMBDA*BNU : K2=K2+1
1530     IF K2>10 GOTO *END1
1535     PRINT "     J=";K2,"lambda=";AMBDA
1540     GOTO *REPEAT2
20000 *INIT1
20010  DEFINT I-N
20020  READ M
20030  DIM X(M),IR(M),IR2(M),P(M),AN(M)
20040  IR2(0)=0 : IRSUM=0
20050  FOR I=1 TO M
20060     READ X(I)
20080  NEXT I
20090  FOR I=1 TO M
20100     READ IR(I)
20110     IRSUM=IRSUM+IR(I) : IR2(I)=IRSUM
20120  NEXT I
20200 RETURN
20500 *PRIOUT
20510  FOR I=1 TO M
20520     PRINT X(I),IR(I),IR2(I)
20530  NEXT I
20540 RETURN
```

```
21000 *YVALUE
21010  Y=0
21020  FOR I=1 TO M
21030     AN1=AN(I) : P1=P(I) : R1=IR(I)
21040     Y=Y+(R1-AN1*P1)*(R1-AN1*P1)/(AN1*P1*(1-P1))
21050  NEXT I
21060 RETURN
22000 *DIFFER
22010  A=0 : B=0 : D=0 : E=0 : F=0
22020  FOR I=1 TO M
22030     AN1=AN(I) : AN2=AN1*AN1 : AN3=AN1*AN2
22040     P1=P(I) : P2=P1*P1 : P3=P1*P2
22045     Q1=1-P(I) : Q2=Q1*Q1 : Q3=Q1*Q2
22050     R1=IR(I) : R2=R1*R1
22060     X1=X(I) : X2=X1*X1
22070     RNP1=R1-AN1*P1 : RNP2=R1+AN1*P1
22080     E=E+RNP1*RNP2/AN2/P1/Q1
22090     F=F+X1*RNP1*(RNP2-2*R1*P1)/AN1/P2/Q2
22100     A=A+R2/AN3/P1/Q1
22110     B=B+X1*(R2*(1-2*P1)+AN2*P2)/AN2/P2/Q2
22120     D=D+X2*(R2*(1-3*P1+3*P2)+AN1*P3*(AN1-2*R1))
            /AN1/P3/Q3
22130  NEXT I
22140  A=2*A : D=2*D
22150 RETURN
23000 *SOLUB
23020  BUNBO=A*D-B*B
23030  DELTAN=(E*D-B*F)/BUNBO
23040  DELTAP=(A*F-E*B)/BUNBO
23050 RETURN
24000 *OUTPUT
24010  PRINT "K=";K1
24020  PRINT "p=";POLD,"n=";ANOLD,"Y=";Y
24030  PRINT "               lambda=";AMBDA
24100 RETURN
25000 *END1
25010  PRINT "Completed!"
25020 END
26000 *END2
26010  PRINT "Not completed?"
26020 END
30000 '---------------------------------------
30005 '    Number of data
30010         DATA 5
30020 '    Units of effort (xi)
30030         DATA 7, 5, 10, 8, 4
30040 '    Size of i th sample (ri)
30050         DATA 700, 465, 884, 636, 293
30060 '    Initial value of p and n
30070         DATA .02 , 20000
30100 '---------------------------------------
```